Optimal Resampling of Finite Energy Signals

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Abstract In this paper, we present a consistent resampling scheme for the non-bandlimited discrete sequences. The consistency principle has been introduced in approximation systems with non-ideal sampling functions. We extend the concept to a resampling system such that the characteristics of the sequence are optimally preserved for the given interpolation function. The consistent resampling can be achieved by inserting a correction filter into the system. Without the presence of noise, the filter can be derived straightforwardly. For a system affected by noise, the problem can be formulated as a semi-definite program (SDP). We show experimentally that the consistent resampling scheme achieves outstanding result in many applications, including image processing such as zooming, rotation, and noise removal. It is also applied in the channel detection using Pilot Symbol Assisted Modulation (PSAM) scheme.

Keywords signal resampling, correction filter, error analysis.

1. INTRODUCTION

In Digital Signal Processing (DSP) system, a resampling process produces a discrete output signal for a given discrete input signal with ideally the same characteristics. For example, a resampled image is produced when we zoom in or out a digital image or when we rotate the image [1]. In some cases an analog signal can also be viewed as discrete if it is impulsive in time domain [2]. For instance, the transmitted signals of an ultra-wideband (UWB) impulse radio system can be considered as a Dirac pulse train and thereby treated in the discrete domain mathematically. The receiver can then be viewed as a resampling filter.

The conventional sampling system uses the classic *Diracs-sinc* pair to sample and reconstruct a continuous signal. However, its application is restricted to signals of bandlimited response. This limitation is over passed if non-ideal sampling functions are adopted and then reconstructed accordingly [3]. When the sampling and approximation functions form dual functions, the reconstructed signal is a projection of the signal onto the space covered by the sampling function. When non-dual operators are used, the concept of *consistency* is introduced to compensate for the non-idealness [4-5]. This scheme has been successfully applied in many interpolation systems. However, when applied directly to the resampling system, contradictory results arise [6]. The authors thereby argued that while interpolation system describes a $L^2 \to L^2$ process, the resampling process is an operation from $\ell^2 \to \ell^2$. The indifferent treatment between resampling and interpolation results in avoidable performance deterioration for the resampling system.

In this paper, we restate the principle of consistency for the resampling system and therein develop a high-performance resampling scheme. We consider the resampling system with and without the presence of noise. The performance is assessed by a new distance metric proposed by the authors recently [7]. When the resampling is noise free, a correction filter can be derived straightforwardly. When noise is present, we aim to minimize the maximum error to ensure the point-wise performance. The correction filter can then be found as a solution of a Semi-Definite Program (SDP) problem. There are two outstanding features about our system. First, it is applicable to non-ideally sampled sequences, i.e., those obtained by using functions other than impulse train. Therefore, the bandlimited constraint is removed and the high frequency components can be effectively preserved. Second, by adopting the consistency principle, we ensure optimal performance for the given interpolation function.

The rest of this paper is organized as follows. In Section 2, we introduce the principle of consistency and restate it for the resampling system. A correction filter is designed to assist the noiseless consistent resampling system. In Section 3, we introduce the performance metric and in Section 4, we continue to analyze the resampling system based on this metric. The correction filter is designed to minimize the maximum error in Section 5. In Section 6, we show through examples the effectiveness of our technique in image application and communication channel modeling.

2. PRINCICPLE OF CONSISTENCY

When non-dual sampling and reconstruction function are used in an approximation system, perfect reconstruction can no longer be obtained. Nevertheless, the consistent principle looses the requirement and states that the reconstructed signal should appear identical to the sampling function as the original input signal. In other words,

$$\left\langle f(x), \varphi\left(x/T - n\right)\right\rangle = \left\langle \tilde{f}(x), \varphi\left(x/T - n\right)\right\rangle \quad \forall n \qquad (1)$$

where $\varphi(x)$ is the sampling function, T is the sampling step, $f(x)$, $\tilde{f}(x)$ are the signal in the Hilbert Space H and its approximation respectively. It has been shown that the consistent reconstruction scheme performs optimally by achieving the minimum mean square error $\left\| f - \tilde{f} \right\|_{L^2}$, for all $\tilde{f} \in V$, $V = span\left\{\varphi\left(x/T - n\right)\right\}_{n \in Z}$.

Different from the interpolation system, the resampling system is to evaluate a sequence at positions where it is unknown previously. A typical resampling system is shown in Fig. 1. We shall denote a discrete signal as $\overrightarrow{f_a} = [f_a[0], f_a[1], \cdots f_a[n-1]] \in C^n$ where the subscript indicates the sampling interval. $\vec{f}_a$ is interpolated by ϕ and $\bar{f}(x) = \sum_n f_a[n] T_a^n \phi$ where $T_a^n \phi = \phi\left(x/a - n\right)$. This can alternatively be represented more compactly as $\bar{f}(x) = \overrightarrow{\phi_a} \overrightarrow{f}_a$ where $\overrightarrow{\phi_a} : C^n \to \mathrm{H}$ consists of a set of vectors ϕ_a^i such that $\phi_a^i = T_a^i \phi$ for $1 \le i \le n$. Similarly, the sampling process $\overrightarrow{f_b} = \overrightarrow{\varphi_b}^* \bar{f}(x)$ is defined through the adjoint operation $\overrightarrow{\varphi_b}^* : \mathrm{H} \to C^m$ with $\varphi_b^i = T_b^i \varphi$ for $1 \le i \le m$. It follows that $f_b[i] = \left\langle \bar{f}(x), T_b^i \varphi \right\rangle$.

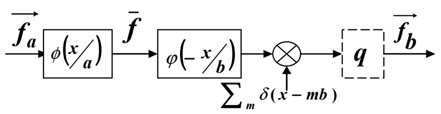


Figure 1: Block diagram of a resampling system.

For a given resampling system, it is of high probability that the interpolation function is non-ideal. For example, when $\vec{f}_a$ is obtained by ideal sampling $f_a[n] = f(x)\big|_{x=na}$, the ideal interpolation function is the $\phi(x) = \mathrm{sinc}(x/a)$. However, due to its slow convergence rate, it is never used in practical applications. Instead, functions of local support such as spline are employed. It is reasonable, then, to apply the consistency principle to the resampling system for non-dual sampling and to involve interpolation functions. We apply the principle of consistency for the resampling system as a whole and restate it as:

Definition 1: A resampling system is consistent if the output sequence approximates the same analog signal as the input sequence through a properly dilated interpolating function. Referring to Fig. 1, the resampling is consistent if

$$\sum_n f_a[n] T_a^n \phi = \sum_m f_b[m] T_b^m \phi \qquad (2)$$

A correction filter can be inserted into the system to enforce the consistently resampling. We summarize the result in Proposition 1:

Proposition 1: For the resampling system as shown in Fig. 1, denote the dual function of $\phi\left(x/b\right)$, $\varphi\left(x/b\right)$ by $\phi_d\left(x/b\right)$ and $\varphi_d\left(x/b\right)$ respectively. If they exist, consistent resampling can be achieved by inserting a correction filter whose response is given by

$$Q\left(\omega/b\right) = \Phi_d(\omega) \Psi_d(\omega) \qquad (3)$$

Proof: Let $\tilde{f}(x) = \sum_m f_b[m] T_b^m \phi$. Referring to Fig. 1, it can be rewritten as

$$\tilde{f}(x) = \vec{\phi}_b \vec{\varphi}_b^* q \bar{f}(x) \qquad (4)$$

Take samples of $\tilde{f}(x)$ using $\phi_d\left(x/b\right)$, and interpolate the resultant sequence using $\varphi_d\left(x/b\right)$,

$$(\vec{\varphi}_b)_d \left(\vec{\phi}_b\right)_d^* \tilde{f}(x) = q\bar{f}(x) \qquad (5)$$

Inasmuch as for dual function pairs we have $\left(\vec{\phi}_b\right)_d^* \vec{\phi}_b = 1$ and $(\vec{\varphi}_b)_d \vec{\varphi}_b^* = 1$. For a correction filter whose response is specified as in (3), $\tilde{f}(x) = \bar{f}(x)$ and the resampling is consistent.

3. PERFORMANCE METRIC IN DISCRETE SPACE

For interpolation system, the performance of approximation can be assessed by $\left\| f - \tilde{f} \right\|_{L^2}$, and the norm of the difference is defined in H. Unfortunately, the counterpart norm definition in ℓ^2 does not constitute a proper performance metric for the resampling system. To address this problem, a novel distance metric for sequences in ℓ^2 is recently proposed by the authors, which is restated here for the sake of completeness:

Definition 2: Given the resampling system in Fig. 1, the distance between $\overrightarrow{f_a}$ and $\overrightarrow{f_b}$ is defined by

$$D_\phi\left(\overrightarrow{f_a},\overrightarrow{f_b}\right) = \left\|\overrightarrow{\phi_a}\overrightarrow{f_a} - \overrightarrow{\phi_b}\overrightarrow{f_b}\right\|_{L^2} \quad (6)$$

This definition satisfies the properties of a metric and is strictly non-negative. However, it is a pseudo metric since when $D_\phi\left(\overrightarrow{f_a},\overrightarrow{f_b}\right)=0$, $\overrightarrow{f_a}$ and $\overrightarrow{f_b}$ can be two different sequences. For a comprehensive discussion on the properties of the metric, please refer to [7-9].

One property that worth mentioning here is that when $D_\phi\left(\overrightarrow{f_a},\overrightarrow{f_b}\right)=0$, the resampling is lossless in terms that we can recover the input from the output.

***Proposition 2*:** Let $c_a^\phi = \overrightarrow{\phi_a^*\phi_a}$ be the autocorrelation function of ϕ_a. If $D_\phi\left(\overrightarrow{f_a},\overrightarrow{f_b}\right)=0$ and c_a^ϕ is invertible, $\overrightarrow{f_a}$ can be reconstructed from $\overrightarrow{f_b}$.

Proof: Since $D_\phi\left(\overrightarrow{f_a},\overrightarrow{f_b}\right)=0$, $\overrightarrow{\phi_a}\overrightarrow{f_a} = \overrightarrow{\phi_b}\overrightarrow{f_b}$. To reconstruct $\overrightarrow{f_a}$ is to find the set of coefficients such that $\vec{\phi}_a\vec{f}_a = \bar{f}(x)$. This is equivalent to passing $\hat{f}(x)$ through the dual operator of ϕ_a. If c_a^ϕ is invertible, the dual operator is specified in the frequency domain by

$$(\Phi_a)_d = \Phi_a / C_a^\phi \quad \textbf{(7)}$$

where Φ_a, C_a^ϕ are the Fourier transforms of ϕ_a and c_a^ϕ respectively and $C_a^\phi(\omega) = \sum_n \left|\Phi(\omega + 2na\pi)\right|^2$. Thus, $\overrightarrow{f_a}$ can be reconstructed by $\vec{f}_a = \vec{h} * \bar{f}$, $h(x) = (\phi_a)_d(-x)$.

4. NOISY RESAMPLING

Additive noise can be introduced into the resampling system in two different ways. First, the input signal may be corrupted before it is interpolated, as shown in Fig. 2. Alternatively, as the second way, noise may be considered to be added to the interpolated signal in the continuous domain as shown in Fig. 3. In this section, we derive the equations of the mean squared error (MSE) performance of the resampling system under these two types of noise.

4.1. Noisy Resampling System with Discrete Noise

Referring to Fig. 2, by assuming that the noise is of the same length as $\vec{f}_a$, the newly reconstructed signal is given by

$$\bar{f}'(x) = \overrightarrow{\phi_a}\overrightarrow{f_a} + \overrightarrow{\phi_a}\overrightarrow{N} \quad (8)$$

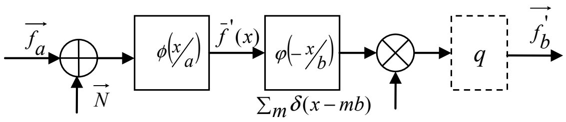


Figure 2: Noisy resampling system -- discrete noise added to the input signal.

The output is specified as

$$\overrightarrow{f_b'} = \overrightarrow{\varphi_b^*}q\bar{f}'(x) \quad (9)$$

The distance between $\vec{f}_a$ and $\vec{f}_b'$ is given by

$$D_\phi\left(\vec{f}_a,\vec{f}_b'\right) = \left(I - \overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}q\right)\bar{f} - \overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}q\overrightarrow{\phi_a}\overrightarrow{N} \quad \textbf{(10)}$$

Let us assume that the noise is a zero-mean with positive-definite covariance matrix V. The average performance is given by the expected value of the MSE of the system:

$$\begin{aligned} E\left[MSE_d(\vec{f}_a,q)\right] &= E\left[D_\phi^2\left(\vec{f}_a,\vec{f}_b'\right)\right] \\ &= \left\|\left(I - \overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}q\right)\bar{f}\right\|^2 + Tr\left(\overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}\right)^*\left(\overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}\right)qV\left\|\phi_a\right\|_{L^2}^2 q^* \end{aligned} \quad (11)$$

The MSE of the resampling system depends not only on the correction filter, but also on the input sequence, the interpolating function as well as the noise. This expression becomes input independent if we let $I = \overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}q$. Thus the first term can be removed. Hence the correction filter is given by $q = \left(\vec{\phi}_b\vec{\varphi}_b^*\right)^{-1} = (\vec{\varphi}_b)_d(\vec{\phi}_b)_d^*$, which is consistent with Proposition 1 for the noiseless resampling system.

4.2. Noisy Resampling System with Continuous Noise

In the case of continuous noise as shown in Fig. 4, the output of this system is given by

$$\overrightarrow{f_b''} = \overrightarrow{\varphi_b^*}q\left(\bar{f}(x) + n(x)\right) \quad (12)$$

where the noise $n(x)$ is assumed to be a stationary zero-mean process with variance σ.

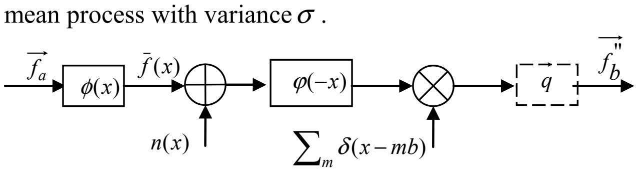


Figure 3: Noisy resampling system -- continuous noise added after interpolation

The average MSE is given by

$$E\left[MSE_c\left(\vec{f}_a, q\right)\right] = E\left[D_\phi^2\left(\vec{f}_a, \vec{f}_b''\right)\right]$$
$$= \left\|\left(I - \overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}q\right)\vec{f}\right\|^2 + \left(\overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}\right)^*\left(\overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}\right)q\sigma q^* \tag{13}$$

To eliminate the dependency on input signals, we observe that the same correction filter developed in Proposition 1 should be employed.

However, this unbiased filter may not lead to a small MSE at all. As we will show in Section 5, for finite energy signals, i.e. $\left\|\vec{f}_a\right\|_{\ell^2} < \infty$, there exists a filter q' which results in a smaller average MSE than the unbiased filter q.

5. OPTIMAL MIN-MAX RESAMPLING

In [10-12], the performances of correction filters subject to various metrics for the sampling/reconstruction system in the presence of noise are provided. The metrics considered include the least square, regularized least squares, minimax MSE, etc. One result obtained from these studies is that among all the filters, the minimax solution gives the best possible performance since it minimize the MSE in the worst case. Therefore we shall concentrate on solving the following minimax problem:

$$q' = \arg\min_Q \max_{\vec{u}\in U}\left\{E\left[MSE(\vec{u}, Q)\right] - E\left[MSE(\vec{u}, q)\right]\right\} \tag{14}$$

We denote the set of discrete signal sequences satisfying the norm bounded (finite energy) condition by U, where $U = \left\{\left\|\vec{u}\right\|_{\ell^2} \le B, B < \infty\right\}$. Among all possible correction filters Q, we want to find $q' \in Q$ which minimizes the average MSE for all $\vec{u} \in U$.

5.1. Minimax MSE Solution for Discrete Noise

Substituting (11) into (14), we have

$$q' = \arg\min_Q\left\{Fn(Q) + \max_{\vec{u}\in U} Fn(\vec{u})\left\|\overrightarrow{\phi_a}\vec{u}\right\|_{\ell^2}^2\right\} \tag{15}$$

where

$$Fn(Q) = Tr\left(\overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}\right)^*\left(\overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}\right)QV\left\|\phi_a\right\|_{L^2}^2 Q^* \tag{16}$$

$$Fn(\vec{u}) = \left(I - \overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}Q\right)^*\left(I - \overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}Q\right) - P \tag{17}$$

$$P = \left(I - \overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}q\right)^*\left(I - \overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}q\right) \tag{18}$$

Here we removed the term $Tr\left(\overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}\right)^*\left(\overrightarrow{\phi_b}\overrightarrow{\varphi_b^*}\right)qV\left\|\phi_a\right\|_{L^2}^2 q^*$ since it is not involved in the minimization process. Let $M = \left\|\phi_a\right\|_{L^2}$. Then by the Schwartz inequality, $\left\|\vec{f}(x)\right\| \le MB$. Let $\vec{r}$ and $\overrightarrow{r^*}$ be the orthonormal set transformation for the space V_a and let $\vec{d} = \overrightarrow{r^*}\vec{f}(x)$, $\left\|\vec{d}\right\| \le MB$. Furthermore, let $\vec{s} = \overrightarrow{r^*}\overrightarrow{\phi_b}$ and $\vec{t} = \overrightarrow{\varphi_b^*}Q\vec{r}$, then the minimization problem (15) becomes

$$\min_Q\left\{Tr\left[\overrightarrow{st}V\left\|\phi_a\right\|_{L^2}^2\overrightarrow{s^*}\overrightarrow{t^*}\right] + \max_{\left\|\vec{d}\right\|\le MB} Fn(\vec{d})\right\} \tag{19}$$

Let $Z = \overrightarrow{r^*}(I - \overrightarrow{st})^*(I - \overrightarrow{st})\vec{r} - \overrightarrow{r^*}P\vec{r}$ and $Fn(\vec{d})$ is in the form $Fn(\vec{d}) = \overrightarrow{d^*}Z\vec{d}$. The second term in (19) is to equivalent to finding the largest eigenvalue of Z, which is the solution of

$$\min_{\lambda\ge 0}\left\{\lambda : Z \prec= \lambda I\right\}$$

Thus, (19) becomes

$$t' = \arg\min_{\vec{t},\lambda\ge 0}\left\{Tr\left[\overrightarrow{st}VM^2\overrightarrow{t^*}\overrightarrow{s^*}\right] + (MB)^2\lambda\right\} \tag{20}$$

subject to

$$(I - \overrightarrow{st})^*(I - \overrightarrow{st}) - P \prec= \lambda I \tag{21}$$

A solution for t' can be found by using the Semi-definite Programming (SDP) techniques [12-13]. The optimal correction filter is given by

$$q' = \left(\vec{r}\right)^{-1} t'\left(\overrightarrow{\varphi_b^*}\right)^{-1} \tag{22}$$

5.2. Minimax MSE Solution for Continuous Noise

Using the same approach for the continuous noise case, the optimal q' can be found by solving the SDP problem

$$w' = \arg\min_{\vec{w},\lambda\ge 0}\left\{\sigma\overrightarrow{w}\overrightarrow{y}\overrightarrow{y^*}\overrightarrow{w^*} + (MB)^2\lambda\right\} \tag{23}$$

subject to

$$\left(I - \overrightarrow{w}\overrightarrow{y}\right)^*\left(I - \overrightarrow{w}\overrightarrow{y}\right) - P \prec= \lambda I \tag{24}$$

where $\vec{w} = \overrightarrow{r^*}q\overrightarrow{\phi_b}$, $\vec{y} = \overrightarrow{\varphi_b^*}\vec{r}$. The optimal solution is

$$q' = \left(\overrightarrow{\phi_b}\right)^{-1} w'\left(\overrightarrow{r^*}\right)^{-1} \tag{25}$$

There are two points we want to highlight. First, M is a real number and can be removed from (20), while in (22), it acts as a regulating factor. This is because the discrete noise is interpolated and the effect of the noise is amplified in the same way as the input noiseless signal. However for the continuous case, the noise is added after interpolation, hence the interpolating function will affect the input signal only. Second, if the sampling function is the Dirac delta train function, the continuous noise is sampled and added to the resampled sequence $\overrightarrow{f_b} = \overrightarrow{\varphi_b}q\overrightarrow{\phi_a}\overrightarrow{f_a} + \overrightarrow{n_b}$ and the correction filter can be obtained by solving the linear regression problem.

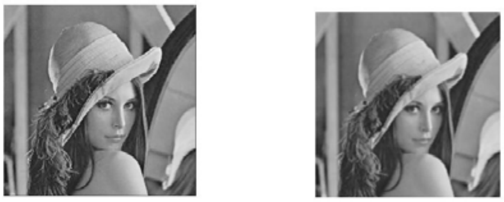
6. EXAMPLES

The first example shows that the consistent resampling outperforms consistent sampling algorithms when it is applied to image applications.

Example 1: The images are zoomed in 8 times by 1.25, and then zoomed out by 0.8 repeatedly for 8 times. The interpolating function is chosen to be linear spline $\beta^1(x)$, where $\beta^1(x) = 1 - |x|$ for $|x| \le 1$ and 0 otherwise. In the first case, $a = 1$, $b = 0.8$ and the correction filter is given by

$$Q(z) = \left(\frac{1}{7} z^{-1} + \frac{5}{7} + \frac{1}{7} z \right)^{-1} \qquad \textbf{(26)}$$

Fig. 4 shows the output image produced by (a) consistent resampling and (b) consistent interpolation respectively. It is obvious that the high frequency components are better reserved (details of hair, hat edge). Other classic images are also tested and the results are stored in Table 1. It is shown that the consistent resampling algorithm outperforms the interpolation algorithm for all images.



(a) (b)

Figure 4: Image zooming using (a) consistent resampling and (b) sampling.

TABLE 1: PSNR results for image zooming applications

PSNR	Consistent Resampling	Consistent Sampling
Lena	65.93	61.21
Rays	26.87	19.71
Peppers	65.25	58.62
Head	53.20	50.58

The effectiveness of our noisy resampling scheme can be illustrated through two examples.

Example 2: Fig. 5 shows how an image corrupted by noise is recovered using the correction filter. In this example the objective is to remove the noise and therefore the sampling rate does not change i.e. $a = b = 1$. The interpolator used is the first order B-spline β_1 and thus $M = 2/3$. A Gaussian noise of zero mean and variance 0.005 is added to the image. From [15] we know that the integer translates of a B-spline form a Riesz sequence in $L^2(R)$, thus the orthonormal vectors spanning the same space is given by:

$$R(\omega) = \frac{B_2(\omega)}{\left(\sum_{-\infty}^{\infty} \left| B_2(\omega + 2k\pi)^2 \right| \right)^{1/2}} \qquad (26)$$

Assume that all the pixel values are in the range $[0,1]$. Therefore B depends only on the size of the image. The correction filter is applied to each row of the image. Fig. 5(a) shows the result of the correction filter resampling approach. This can be compared with Wiener filtering with neighborhood size set to 2×2, as shown in Fig. 5(b). As is obvious from the figures, our correction filter performs better than the Weiner filter. While the Wiener filter is optimal for Gaussian noise, it cannot compensate for a non-ideal synthesis filter (interpolator) which is β_1 in this case.

(a) Recovered by q'

(b) Recovered by Wiener filter

Figure 5: Image recovered from Lena being corrupted by Gaussian Noise of zero mean and variance 0.005

Example 3: We consider Pilot Symbol Assisted Modulation (PSAM) [16], where the pilot symbols are multiplexed into the data symbols to form a series of data frames. Each data frame contains one pilot symbol and $L-1$ data symbols. During transmission, the modulated signal is corrupted by noise. At the receiver side, the channel responses at the positions of pilot symbols in frame k are obtained, denoted by $h_{p,k}$. To estimate the response at the data bits positions $h_{l,k}$, $h_{p,k}$ are interpolated and resampled. Assume that the noise zero-means Gaussian with unit variance. The signal-to-noise ratio (SNR) is simply the pilot symbol energy, or its l^2 norm. Assume that the channel response is bandlimited and the interpolating function is $\phi(x) = \text{sinc}(x/W)$, where W is the channel bandwidth. Due to the slow decaying property of sinc, only $h_{p,k}$ from the nearest K data frame are used in the interpolation process. The resampling function is $\varphi(x) = \delta(x)$. Let $a = 1$ and the resampling rate is $b = 1/L$. Assume that the modulation scheme used is PSK. Fig. 6 shows the Symbol Error Rate (SER) for different SNR. Compared to interpolation schemes using Hamming windowed sin c function, our correction filter generates lower SER for the same SNR.

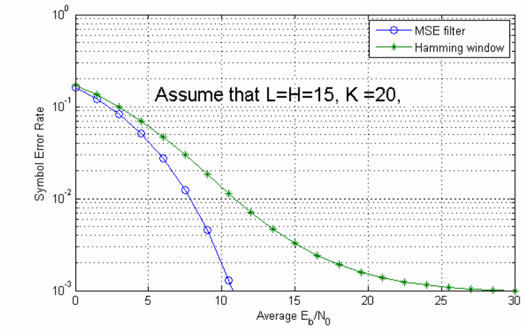


Figure 6: The performance of communication by modeling channel using MSE filter and Hamming window

7. CONCLUSIONS

In this paper, we have applied the principle of consistency to the resampling system. We considered the systems with and without and presence of noise. Consistent resampling can be achieved by inserting a correction filter into the system. For noiseless system, the filter can be derived straightforwardly. When noise is present, we formulate a SDP problem which minimizes the maximum error. The correction filter is then worked out as a solution to the SDP problem. The consistent resampling scheme has been applied to image denoising and PSAM digital communication systems to show that it performs better than the existing techniques.

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