Flock Diameter Control in a Collision-Avoiding Cucker-Smale Flocking Model

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Abstract. Both the original Cucker-Smale flocking model and a more recent version with collision avoidance do not have any control over how tightly the system of agents flock, which is measured by the flock diameter. In this paper, a cohesive force is introduced to potentially reduce the flock diameter. This cohesive force is similar to the repelling force used for collision avoidance. Simulation results show that this cohesive force can reduce or control the flock diameter. Furthermore, we show that for any set of model parameters, the cohesive force coefficient is the single determining factor of this diameter. The ability of this modified collision-avoiding Cucker-Smale model to provide control of the flock diameter could have significance when applied to robotic flocks.

Keywords: Flock diameter control, flocking, Cucker-Smale model, collision avoidance

1 Introduction

Swarm Intelligence is inspired by biological swarms with emergent behaviours that evolve to collectively solve a problem. It has found applications in a diversity of areas [4,7,10]. One of these collective behaviours is flocking which is a phenomenon where individual autonomous agents use only limited information to self-organize into a state of motion consensus, starting from a disordered initial state [15]. In [11], three simple rules — separation, alignment and cohesion, are used to simulate flocking behaviour. The separation rule keeps the agents from colliding. The alignment rule helps them to reach a common speed and direction, while the cohesion rule keeps the flock together spatially. They have been used successfully for computer animation of flocks of birds, etc. However, this model is not amenable to mathematical analyses.

The first mathematical flocking model was proposed by Vicsek [14]. With this model, a group of self-propelled particles moves at the same speed but initially at random directions. Each particle updates its direction by averaging those of its neighbours within a certain radius. Based on the Vicsek's model, Cucker and Smale [3] later proposed a flocking model governed by the following equations:

$$\begin{cases} \dot{p_i} = v_i \\ \dot{v_i} = \frac{1}{N} \sum\limits_{j=1}^{N} \psi\left(\|p_j - p_i\|\right)\left(v_j - v_i\right) \end{cases} \tag{1}$$

for N agents where $1 \leq i \leq N$ and the position and velocity of the i-th agent are denoted by p_i and v_i respectively. The communication rate function ψ quantifies the influence between i-th and j-th agents. It is a positive decreasing function of the Euclidean distance between the agents. With

$$\psi(\|p_j - p_i\|) = \frac{1}{(1 + \|p_j - p_i\|^2)^{\beta}}, \tag{2}$$

it has been mathematically proven that when $\beta < 1/2$ flocking will emerge unconditionally, while for $\beta \geq 1/2$ flocking could only be guaranteed under some conditions on the initial positions and velocities of particles [3]. More recently, a generalization to the Cucker-Smale model has been proposed to ensure that the agents do not collide [1, 2]. This is achieved by adding a repelling force function f such that the model equations become

$$\begin{cases} \dot{p_i} = & v_i \\ \dot{v_i} = & \frac{1}{N} \sum_{j=1}^{N} \psi(\|p_j - p_i\|)(v_j - v_i) \\ & + \sum_{j \neq i} f(\|p_j - p_i\|^2)(p_j - p_i) \end{cases} \tag{3}$$

This is a more realistic model in practice.

The remaining issue is related to the flock diameter. This is defined the maximum distance between any two agents in the flock [6]. In previous studies such as [5, 8, 13], flocking is assumed to be achieved when the velocity is aligned and this diameter attains any finite value, no matter how large. Intuitively, we only use the term flocking to a group of agents that are moving reasonably close to each other. However, with the Cucker-Smale system, there is no control over the final flock diameter. In practice, for example, in the deployment of a group of autonomous robots, we often want to be able to exert some control over the flock diameter. The main aim of this paper is to propose a modified model based on (3) that will allow us more control over the flock diameter. Inspired by the idea of the repelling force for collision avoidance, we introduce a cohesive force in order to achieve this. The effectiveness of this modified model is demonstrated through computer simulation and the relationship between flock diameter and the cohesive force parameter is obtained.

The rest of this paper is organized as follows. A definition of flocking and a description of the repelling force function of collision-avoiding Cucker-Smale system are presented in Section 2. In Section 3.1, our proposed introduction of a a general cohesive force function to the collision-avoiding Cucker-Smale model is discussed. The effects of this model on the flock diameter are studied through computer simulation and the results are presented in Section 4. Finally, Section 5 concludes the paper and discuss the future work by using nonlinear control methods.

2 Preliminaries

2.1 Definition of Flocking

Flocking is said to be achieved for a group of N agents if the following two conditions are satisfied:

1. The velocity of every agent is virtually the same, i.e. for an arbitrarily small $\delta > 0$,
$$|v_i - v_j| \leq \delta \tag{4}$$
for all $i, j \in [1, N], i \neq j$.
An equivalent measure for velocity alignment is the average normalized velocity v_a which is defined by
$$v_a = \frac{\left|\sum_{i=1}^{N} v_i\right|}{\sum_{i=1}^{N} |v_i|} \tag{5}$$
Using this measure, the criterion (4) can alternatively be stated as $|1 - v_a| < \delta'$ for some arbitrarily small $\delta' > 0$.
2. The distance between any two agents is bounded by ϵ. That is,
$$\sup_{1 \leq i,j \leq N} \|p_i - p_j\| < \epsilon \tag{6}$$
ϵ is the upper bound on the distance between two agents that are furthest apart. It will be referred to as the flock diameter in this paper.

2.2 Collision Avoidance

Let $d_0 > 0$ be the minimum distance between any two particles. If $\|p_j - p_i\| < d_0$, then collision is said to have occurred. Collisions could occur between agents in the Cucker-Smale flocking system (1).

In [2], a repelling force is introduced to separate two agents that are too close to each other. It is suggested that this repelling force function $f : (d_0, \infty] \to [0, \infty)$ should have the following properties for $d_1 > d_0$:

1. $\int_{d_0}^{d_1} f(r)dr = \infty$, and
2. $\int_{d_1}^{\infty} f(r)dr < \infty$.

Incorporating this function into (1) results in (3) introduced earlier. A simple function such as $f(r) = (r - d_0)^{-\theta}$ suggested by [2] will have the required properties.

3 Flock Diameter Control

3.1 Cohesive Force

The Cucker-Smale models given by (1) and (3) do not provide any control over the final flock diameter which is the largest distance between any two agents when flocking condition 1 of Section 2.1 is satisfied. Inspired by the way collision avoidance was introduced to (1) through a repelling force, flock diameter could potentially be reduced or controlled by introducing a cohesive force.

This cohesive force function ϕ must be Lipschitz continuous, so that the existence theorems and equations for Cucker-Smale model can apply. Given that a repelling force is asserted when an agent moves within a radius of d_1 of another agent, the cohesive force should operate between agents that are at least at a distance of d_1 apart. So, $\phi(r) = 0$ when $r \leq d_1$. For $r > d_1$, $\phi(r)$ should be a monotonically non-decreasing function of r.

An example of cohesive force function is given by:

$$\phi(r) = \begin{cases} 0 & r \leq d_1 \\ k * \frac{1}{1+e^{-r}} & r > d_1 \end{cases} \tag{7}$$

where $1/(1 + e^{-r})$ is a sigmoid function and is Lipschitz continuous.

3.2 Modified Cucker-Smale System

Introducing this cohesive force into (3), the modified Cucker-Smale system becomes

$$\begin{cases} \dot{p_i} = & v_i \\ \dot{v_i} = & \frac{1}{N}\sum_{j=1}^{N} \psi(\|p_j - p_i\|)(v_j - v_i) \\ & + \sum_{j \neq i} f(\|p_j - p_i\|^2)(p_j - p_i) \\ & + \sum_{j \neq i} \phi(\|p_j - p_i\|^2)(p_j - p_i) \end{cases} \tag{8}$$

for $1 \leq i \leq N$ where ϕ is the cohesive force function.

The second and third terms for $\dot{v_i}$ in (8) could be combined since they essentially have the same form. Hence we have

$$\begin{cases} \dot{p_i} = & v_i \\ \dot{v_i} = & \frac{1}{N}\sum_{j=1}^{N} \psi(\|p_j - p_i\|)(v_j - v_i) \\ & + \sum_{j \neq i} H(\|p_j - p_i\|^2)(p_j - p_i) \end{cases} \tag{9}$$

When the distance between any two particles $r = \|p_j - p_i\|$ is less than collision avoidance distance d_1, then H acts like the repelling force function f. Otherwise, it acts like the cohesive force function ϕ.

Using the examples for the repelling and cohesive forces from Section 2.2 and 3.1 respectively, a possible function H is shown in Figure 1. Here, a positive value denotes an attractive force while negative values denote repulsion.

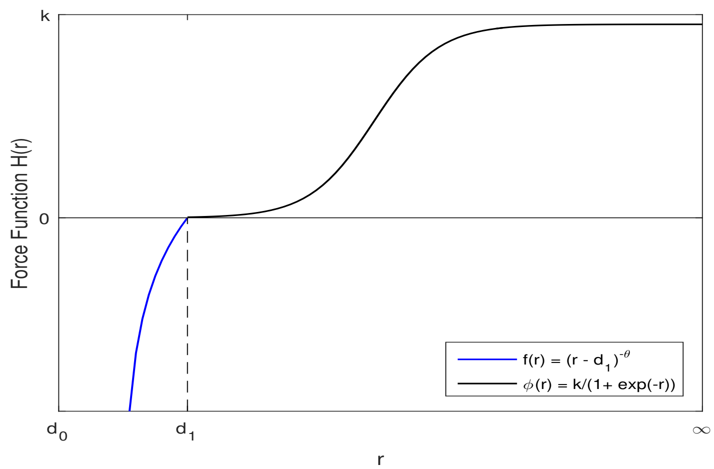

Fig. 1. Force Function $H(r)$ in (9).

4 Simulation Results

The characteristics of the modified collision-avoiding Cucker-Smale system (8) given in Section 3.2 will now be studied using computer simulation. The aim is to evaluate how various parameters of the system affects the flock diameter. These parameters include the cohesive force coefficient k in (7), the initial field size and the collision distance d_1.

The agents are free to move in an infinitely large two-dimensional space. Thus they will not encounter any boundaries. Without loss of generality, we shall not assign units to both the distance and time. Every agent will move with the same speed of 0.5 per unit time with a uniformly random initial direction in $[0, 2\pi)$. The initial position of each agent will be randomly chosen within a circle of l units in diameter which will be referred to as the initial field size. The value of β in (2) is fixed at 1/4 to ensure flocking occurs. The collision avoidance distance d_1 is set at 0.2. The value of d_0 is 0.01. The value of parameter θ of the repelling function is 2. The system is considered to be in a flocking state when the average velocity $v_a \geq 0.99$. In every scenario for each set of parameters, the value shown is the average over 20 independent simulation runs.

4.1 Effect of Cohesive Force Coefficient

First, we shall consider the effect of the cohesive force coefficient k in (7) on the flock diameter when flocking is achieved. We examine values of k between 0 and 3 with $k = 0$ indicating that cohesive force is not used. The size N of the flock ranges from 10 to 50 with increments of 10. The initial field size is 4.

Figure 2 shows that increasing the cohesive force coefficient leads to a substantial reduction in the flock diameter for all different values of N. Comparing

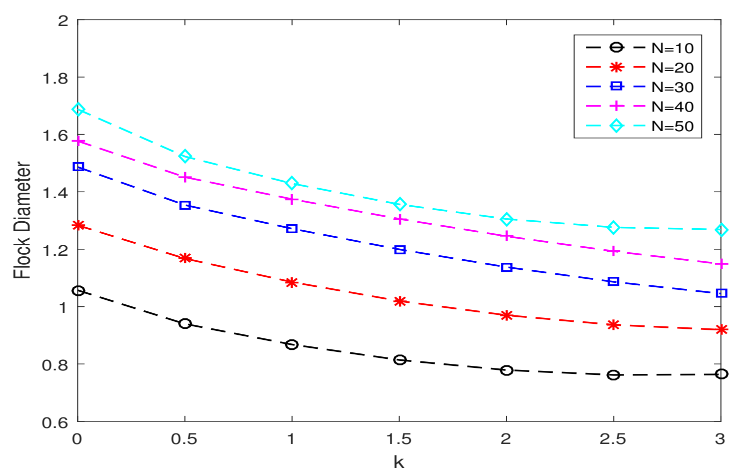

Fig. 2. Flock Diameter for Different Cohesive Force Coefficients

between no cohesive force and $k = 3$ reveals the largest reduction in flock diameter is for $N = 10$ at 29.8%. For N from 20 to 50, percentage reduction in flock diameter are 12.3%, 15.1%, 14.9% and 14.5% respectively. This shows that the cohesive force can be used to obtain a tighter group of flocking agents.

4.2 Effect of Initial Field Size

Table 1. Flock Diameter by Cohesive Force Coefficients (k)

Initial Field Size	N = 10		N = 20		N = 30		N = 40		N = 50	
	k=0	k=1	k=0	k=1	k=0	k=1	k=0	k=1	k=0	k=1
4	1.0570	0.8634	1.2835	1.1080	1.4864	1.2703	1.5771	1.3513	1.6877	1.4280
8	1.5185	1.3644	1.7080	1.6742	1.8716	1.8059	2.0552	1.9180	2.2818	1.9707
12	2.2013	1.7812	2.4258	1.9644	2.6490	2.3418	2.7641	2.4214	2.8702	2.2588
16	2.8674	2.2808	3.0995	2.3925	3.1646	2.4683	3.2858	2.6104	3.3848	2.7377
20	3.0775	2.6609	3.2935	2.8801	3.3419	2.9797	3.4430	3.0980	3.5366	3.1976

The initial field size, which reflects how closely placed the agents initially are, may have a substantial effect on the final flock diameter. In this set of simulations, we vary the intial field size while keeping other system parameters the same. Figure 3 shows that increasing the initial separation of the agents does have a

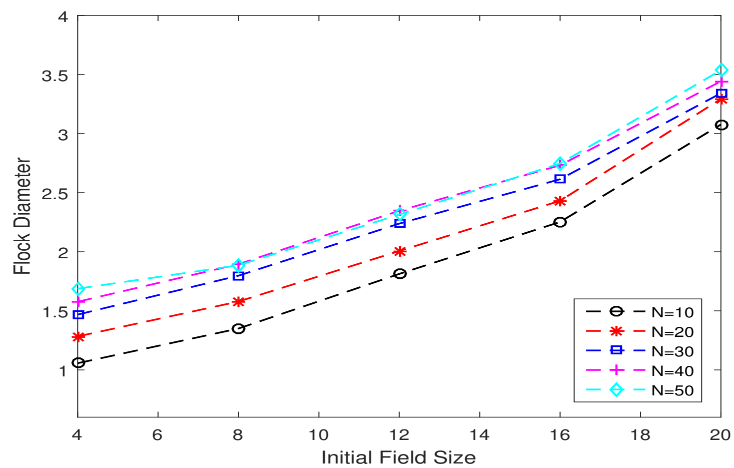

Fig. 3. Flock Diameter for Different Initial Field Size Without Cohesive Force.

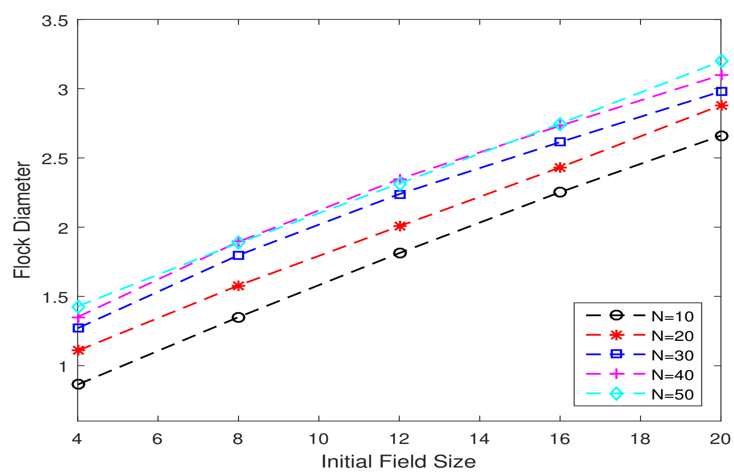

Fig. 4. Flock Diameter for Different Initial Field Size With Cohesive Force Coefficient $k = 1$.

substantial effect on the final flock diameter. For $N = 10$, when the initial field size is increased from 4 to 20, the flock diameter is increased almost 3 times. As N increases, the percentage increase in flock diameter is smaller. But for $N = 50$, the increase is still more than twice.

The same simulations are repeated with a cohesive force coefficient k set to 1. The results can be found in Figure 4. The numerical values for Figures 3 and 4 are listed in Table 1 for ease of comparison. It is interesting to note that the percentage increase in flock diameter with cohesive force is more or less the same that without cohesive force. This is true for all values of N. Thus the effect of initial field size on the final flock diameter is essentially the same for both systems.

4.3 Effect of Collision Distance

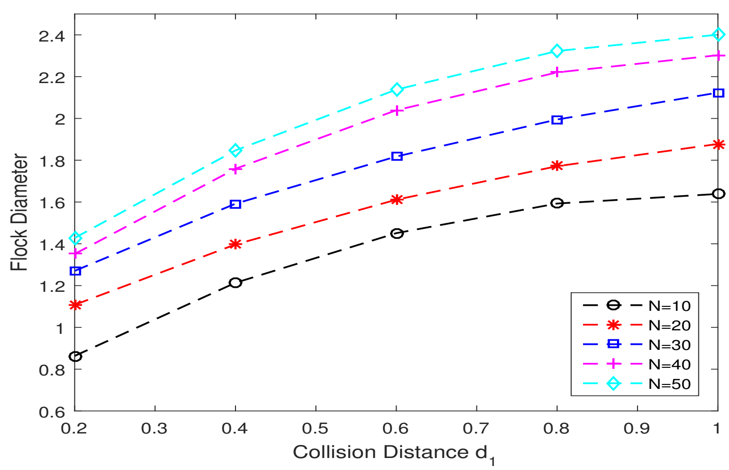

Fig. 5. Flock Diameter in Different Collision Distances

The remaining factor that could have a substantial influence on the flock diameter is the collision distance d_1. Simulation results with $k = 1$ and varying values of d_1 are shown in Fig 5. Other parameters are the same as the previous simulations. The lowest curve shows that collision distance in 1 has approximately 2 times impact on flock diameter when collision avoidance is 0.2 for $N = 10$. Fig 5 displays a similar tendency in the different number of groups. Consider the collision distance $d_1 = 0.2$ as the reference. When d_1 is doubled, the flock diameter increases by 130% for all values of N. For instance, with $N = 50$, the flock diameter is 1.8755 in $d_1 = 0.4$ that is 1.3 times in comparison with 1.4280 of $d_1 = 0.2$. Based on the results from Section 4.1, if d_1 is increased, the only way to reduce the flock diameter is by increasing the cohesive force.

5 Conclusions and Future Work

In this paper, we introduced a cohesive force into the collision-avoiding Cucker-Smale flocking model. The main purpose is to provide a way to control the diameter of the flock. Simulation results show that this cohesive force is able to reduce this diameter, and the cohesive force is obvious a nonlinear control for flocking diameter. Other factors such as the initial field size and collision distance have the same effect on the system with and without cohesive force. Thus the only significant factor in controlling the flock diameter is the cohesive force coefficient. The ability of this modified collision-avoiding Cucker-Smale model to provide control of the flock diameter could have significance when applied to robotic flocks. In the future, we are exploring ideas like [9,12] using nonlinear control for flocking diameters.

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