Design Of Variable Linear Phase FIR Filters Based On Second Order Frequency Transformations And Coefficient Decimation

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***Abstract*— This paper presents the design of a variable linear phase finite impulse response filter based on second order frequency transformations and coefficient decimation. The design of variable digital filters (VDFs) using first and second order frequency transformations have been proposed in literature. The VDF using second order transformation has better cut-off slope characteristics compared to the VDF using first order transformation. However, the former has the drawback of limited range (approximately 25% of the half of the sampling frequency) over which the cut-off frequency, f_c, can be varied. It also fails to provide variable lowpass, highpass, bandpass or bandstop responses from a fixed-coefficient lowpass filter using the same architecture. The architecture proposed here overcomes the above mentioned disadvantages using coefficient decimation technique. The design example shows that the range over which f_c can be varied is 2.65 times wider in the proposed VDF than the VDF in [7] and for a given frequency range, the proposed VDF offers a total gate count saving of 33% and 41% over the VDF in [11] and [7] respectively. Also, the proposed architecture provides variable lowpass, highpass, bandpass or bandstop responses from a fixed coefficient lowpass filter.**

I. Introduction

A variable digital filter (VDF) is a filter whose frequency specifications such as cut-off frequency, f_c, phase delay or group delay can be controlled on-the-fly through a small number of parameters with minimum overhead on complexity. They find applications in multi-standard wireless communication receivers (MWCRs), for channelization and spectrum sensing. For these applications, finite impulse response (FIR) filters are preferred over infinite impulse response (IIR) filters because the former can have exact linear phase with guaranteed stability and low coefficient sensitivity [1]. Here, the discussion is focused on VDF with variable f_c and constant group delay.

The optimal way to change the cut-off frequency, f_c, of a filter is to update all its coefficients. These filters are known as programmable filters [2, 3]. They are appropriate only if the f_c needs to be changed occasionally because the number of coefficients that needs to be updated can be substantial, especially for FIR filters. For applications such as MWCRs where filter coefficients need to be changed much more frequently based on the communication standards, we need digital filter structures where f_c is controlled by a small number of parameters which can be updated with minimal overhead whenever change is needed. Several methods have been proposed to implement such VDFs. The first one is a linear phase VDF based on first order frequency transformation [4]. It is obtained by replacing sub-networks in the Taylor structure of a prototype filter by a sub-network which performs a frequency transformation of the original network. This method is further studied in detail in [5, 6]. In [7], second order frequency transformations are used and the VDFs have the advantage of sharper transition roll-off characteristics for the same multiplication complexity and same number of variable parameters compared with those in [4]. Also, it allows a variation of the f_c both above and below the cut-off frequency of the prototype filter. But the range over which f_c can be varied is limited to approximately 25% of the frequency range of a digital filter.

Another approach for producing direct-form linear phase VDFs with a simple relationship between filter coefficients and f_c, was proposed by Jarske et al. [8]. However, the maximum stopband attenuation achievable by this method is only -40 dB [8]. Recently, few VDFs designs making use of the Farrow structure are proposed [9-11]. In these methods, the overall filter is a weighted linear combination of fixed FIR sub-filters and the weights are polynomial functions of the f_c. This structure admits a simple updating routine and provides good filter performances, but the overall filter complexity is very high. Hence, these methods are not suitable when f_c needs to be varied over a wide frequency range. In addition, these methods [4-11] need to update filter coefficients or filter architecture to obtain variable bandpass or bandstop responses.

In [12], the coefficient decimation (CDM) method for realizing low complexity reconfigurable FIR filters with fixed-coefficients was proposed. This method has the advantage that f_c can be varied over the entire frequency range, i.e. 0 to π in the normalized scale. However, this method is suitable only for designing tunable filters for a finite set of cutoff frequencies.

In this paper, we propose a low complexity VDF based on a combination of second order frequency transformation and the CDM technique. The design example shows that the proposed VDF offers substantial reductions in gate count over other VDFs. The proposed VDF is suitable for serial spectrum sensing and channelization operations in mobile MWCRs due to its low complexity, linear phase and ability to obtain variable lowpass, highpass, bandpass or bandstop responses on-the-fly from a fixed lowpass filter.

The paper is organized as follows. A review of VDF based on second order transformation is presented in Section II. In Section III, proposed VDF is presented. A design example and its implementation results are shown in Section IV and V respectively. Section VI concludes the paper.

II. Variable Digital Filters Based On Second Order Frequency Transformation

Consider a causal linear phase FIR filter, $H(z)$, of order $2N$ with symmetric coefficients (also referred to as the prototype filter). This prototype filter can be implemented in Taylor form by expressing the transfer function as

$$H(z) = \sum_{n=0}^{N} a_n \, z^{-N} \left[\frac{z+z^{-1}}{2}\right]^n \tag{1}$$

where the coefficients a_n are related to the impulse response coefficients h_n of the prototype filter, through the Chebyshev polynomials [4]. Second order transformation is performed by the following substitution [7]:

$$\frac{z+z^{-1}}{2} = \sum_{k=0}^{2} A_k \left(\frac{Z+Z^{-1}}{2}\right)^k \tag{2}$$

where A_k are the transformation coefficients which controls the relationship between the prototype and transformed frequency responses. Substituting (2) in (1), we obtain the transfer function of the transformed prototype filter as

$$H_2(Z) = \sum_{n=0}^{N} a_n \, Z^{-2(N-n)} \underbrace{\left[\sum_{k=0}^{2} A_k Z^{k-2} \left(\frac{1+Z^{-2}}{2}\right)^k\right]^n}_{D(Z)} \tag{3}$$

Let the cut-off frequencies of the prototype and transformed filters be ω_c and Ω_c respectively. From (2), we obtain the relationship

$$\cos\omega = A_0 + A_1 \cos\Omega + A_2 \cos^2\Omega \tag{4}$$

where

$$\cos\omega = \frac{\left(z+z^{-1}\right)}{2}\bigg|_{z=e^{j\omega}} \quad \text{and}$$

$$\cos\Omega = \frac{\left(Z+Z^{-1}\right)}{2}\bigg|_{Z=e^{j\Omega}}$$

From (4), the cut-off frequency Ω_c of the transformed filter is given by

$$\Omega_c = \cos^{-1}\left\{\left\{-A_1 \pm \left[A_1^2 - 4A_2(A_0 - \cos\omega_c)\right]^{1/2}\right\} \Big/ 2A_2\right\} \tag{5}$$

which is solvable for Ω_c if the following constraints are met [7].

$$A_0 + A_1 + A_2 = 1, \quad 0 \le A_1 \le 1, \quad A_2 \le \frac{A_1}{2}$$
$$A_1^2 - 4A_2(1 - A_1 - A_2 - \cos\omega_c) \ge 0 \tag{6}$$

The implementation of the transformed filter, $H_2(z)$, is shown in Fig. 1[4]. The cut-off frequency and roll-off characteristics of the $H_2(z)$ are controlled by the coefficients A_0, A_1 and A_2.

III. Proposed Variable Digital Filter

In [7], the coefficient A_1 in (4) is set to unity in order to reduce the number of multipliers and number of variable parameters. However, it is observed that by fixing $A_1$to unity, the range over which the cut-off frequency can be varied is also limited to approximately 25% of the half of the sampling frequency. We propose a VDF that is able to overcome this drawback using CDM technique and by allowing coefficient A_1 to vary between 0 and 1.

A. Combining with CDM-I

In [12], two techniques, CDM-I and CDM-II, are proposed for the realization of reconfigurable low complexity FIR filters with fixed coefficients. Here, we are using the CDM-I technique to obtain lowpass response with higher

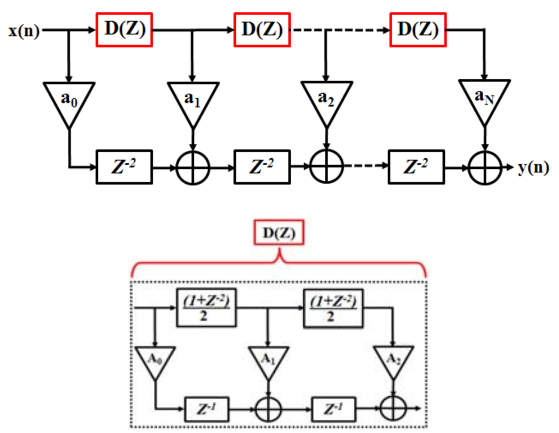

Fig. 1. Variable cut-off linear phase filter using 2nd order transformation.

cut-off frequency from a lowpass response with lower cut-off frequency and vice-versa.

Consider a lowpass prototype filter with frequency response as shown in Fig. 2(a). The CDM-I by a factor of D means that every D^{th} coefficient of the prototype filter is kept unchanged and other coefficients are replaced by zero, resulting in a multiband response. The frequency response obtained from the prototype filter using CDM-I with D=2 is shown in Fig. 2(b). The complementary response of Fig. 2(b) is shown in Fig. 2(c). It is obtained by subtracting the response in Fig. 2(b) from the appropriately delayed version of the input signal. Finally, the lowpass response with higher cut-off frequency is obtained by adding responses in Fig. 2 (a) and (c). In this way, using CDM-I, a higher cut-off frequency response can be obtained from a lower cut-off frequency response without any masking filter and both the responses have same cut-off slope. Thus, the problem of deterioration of cut-off slope at the frequencies away from ω_c in [4-7] is overcome in the proposed VDF.

In the proposed method, CDM-I technique is applied to the VDF discussed in Section II. Then, the cut-off frequency Ω_c of the proposed VDF with CDM-I by factor D = {1, 2} is given by,

$$\Omega_c = \cos^{-1}\left\{\left\{-A_1 \pm \left[A_1^{\,2} - 4A_2(A_0 - \cos(D\omega_c))\right]^{1/2}\right\}\Big/ 2A_2\right\} \quad (7)$$

Thus, the responses in Fig. 2 (a) and 2 (d) are variable lowpass frequency responses. The complementary of the variable lowpass response gives variable highpass response. The variable bandstop and bandpass responses are directly obtained as shown in Fig. 2 (b) and 2 (c) respectively.

B. Variable A_1

The second modification is based on the observation from (7) that the range over which the cut-off frequency Ω_c can be varied depends on ω_c and A_k. To provide maximum flexibility, all the three parameters A_0, A_1 and A_2 should be adjustable. By not restricting the value of A_1 to unity and by using CDM-I technique, our proposed VDF allows a much wider range of cut-off frequencies without increasing implementation complexity and deterioration in cut-off slope at higher frequencies.

The implementation diagram of the proposed VDF is shown in Fig. 3. It is obtained from Fig. 1 with two extra outputs and "adder and MUX" unit. The block, $D(Z)$ is implemented using (2). Let $D(Z)$ be the RHS of (2). Then,

$$D(Z) = A_0Z^{-2} + A_1Z^{-1}\left(\frac{1+Z^{-2}}{2}\right) + A_2\left(\frac{1+Z^{-2}}{2}\right)^2. \quad (8)$$

From the constraints in (6), we know that A_0=1-A_1-A_2. Substituting this relationship into (8) and simplifying, we obtain

$$D(Z) = A_1\left[\left(\frac{Z^{-1}+Z^{-3}}{2}\right) - Z^{-2}\right] + Z^{-2} - A_2\left[Z^{-2} - \left(\frac{1+Z^{-2}}{2}\right)^2\right] \quad (9)$$

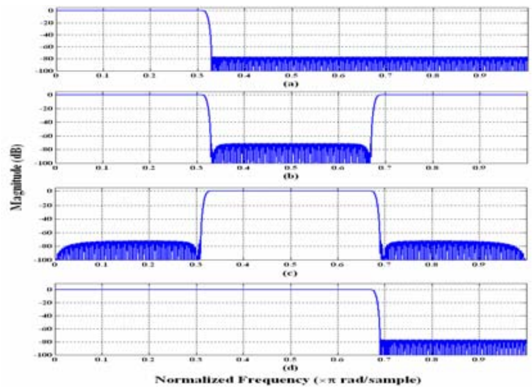

Fig. 2 (a) Frequency response of prototype filter, (b) Frequency response of filter using CDM-I by factor 2 from prototype filter in (a), (c) Complementary response of Fig. 2(b), (d) Frequency response obtained by adding (c) and (a).

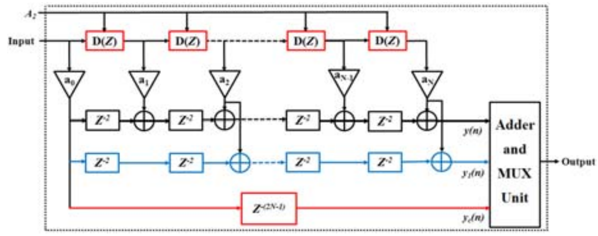

Fig. 3. Proposed VDF.

In this way, only two multipliers are needed instead of three to implement $D(Z)$. The coefficients a_0, $a_1 \ldots a_n$, are fixed and hence can be hardwired. The adder and MUX unit performs the arithmetic operation as discussed before (where $y(n)$ and $y_1(n)$ corresponds to the responses shown in Fig. 2(a) and 2(b) respectively). The variable frequency responses are obtained as follows.

Lowpass response : - $y(n)$ or $[y(n) + y_c(n) - y_1(n)]$

Highpass response : - $[y_c(n) - y(n)]$ or $[y_1(n) - y(n)]$

Bandstop response : - $y_1(n)$

Bandpass response : - $[y_c(n) - y_1(n)]$

IV. DESIGN EXAMPLE

The performance of the proposed VDF is discussed in this section with the help of a suitable design example. Let the cut-off frequency, f_c, and transition bandwidth of the prototype filter be 0.295 and 0.07 respectively. All the frequency edges mentioned here are normalized with respect to half of the sampling frequency. Let the passband and stopband ripple specifications are 0.06 dB and -50 dB respectively. The CDM-I method has an inherent disadvantage of deterioration of stopband attenuation. Hence, the prototype filter needs to be overdesigned. Thus, the order of the prototype filter for the proposed VDF is 88 (=2N) compared to 80 for the VDF in [7].

The variable frequency response ranges for the proposed VDF and VDF in [7] are shown in Fig. 4. For the proposed

VDF with A_1=0.875 and 0.4375 ≤ A_2 ≤ -0.9, the range of f_c is from 0.18 to 0.87. However, the f_c of the VDF in [7] (A_1=1 and -0.5 ≤ A_2 ≤ 0.5) has the limited range from 0.21 to 0.47. Thus, the range of f_c in the proposed VDF is 2.65 times wider than that of VDF in [7]. Also, when VDF in [7] is combined with CDM-I, new range of f_{pass} is from 0.18 to 0.79, which is 2.35 times wider than the original range. The variable bandpass responses for center frequency equal to 0.63 obtained using the proposed VDF is shown in Fig. 5. Similarly, the variable bandpass or bandpass response at different center frequency can be obtained. The group delay of the proposed VDF is constant and hence the proposed VDF have linear phase.

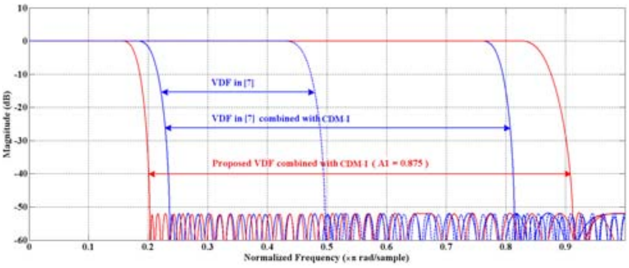

Fig. 4. Variable lowpass responses for proposed VDF and VDF in [7].

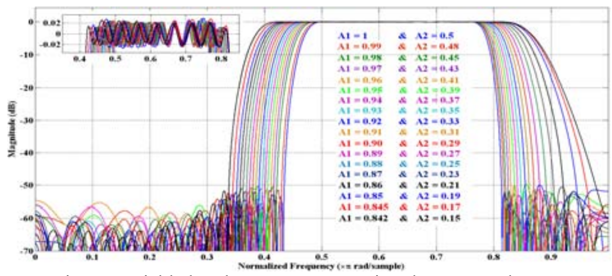

Fig. 5. Variable bandpass responses using the proposed VDF.

V. IMPLEMENTATION COMPLEXITY

In this section, the complexity comparison in terms of total number of gate counts is done. A 16x16 bit multiplier, a 4:1 multiplexer and 32 bit adder were synthesized on a TSMC 0.18μm process. The Synopsys Design Compiler was used to estimate the cell area. The area in terms of gate count, as shown in Table I, was obtained by normalizing the above area values by the cell area of a two input NAND gate from the same library. For the design example discussed above and for the range of f_{pass} from 0.16 to 0.83, our VDF (with A_1=0.875 and hence multiplication with A_1 is replaced with hardwired shifts) offers a total gate count reduction of 33% and 41% over the VDF in [11] and [7] respectively. For the proposed VDF and VDF in [7], transition bandwidth is not fixed over the entire frequency range and varies from 0.04 to 0.12. Hence, for fair comparison, VDF in [11] is designed with TBW= 0.12. Even when multiplication with A_1 is done using general multipliers, the complexity of the proposed VDF is significantly less than other VDFs. Also, other VDFs [4-11] need to update filter coefficients or filter architecture to obtain bandpass or bandstop responses.

TABLE I. COMPLEXITY COMPARISON

VDFs / No. of	Proposed VDF	Proposed VDF (A_1 = 0.875)	VDF in [7] $0.16 \le f_{pass} \le 0.83$	VDF in [11]
Multipliers	133	89	162	196
Adders	378	422	640	172
Multiplexers	2	1	0	0
Total gate count	**311603**	**247193**	**420040**	**372072**

VI. CONCLUSION

The design of linear phase variable digital filter (VDF) using second order transformation and coefficient decimation technique is proposed in this paper. The design example shows that the range over which the cut-off frequency, f_c, can be varied is 2.65 times wider in the proposed VDF than the previous VDF and for a given frequency range, the proposed VDF offers a total gate count saving of 33% and 41% over the VDF in [11] and [7] respectively. The proposed architecture provides variable lowpass, highpass, bandpass and bandstop responses from a fixed coefficient lowpass filter.

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