Restoration of images contaminated by mixed Gaussian and impulse noise using a recursive minimum–maximum method

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Indexing terms: Image restoration, Impulse noise

Abstract: A technique is proposed for removing impulse noise in images, called the recursive minimum–maximum method. Statistical analysis of this method indicates that it is good at preserving fine details and suppressing impulse noise at the same time. Experimental results show that the technique is robust and produces better restored images under various impulse noise conditions than other median filter-based methods.

1 Introduction

Owing to the imperfections of sensors and communication channels, most images are degraded during the processes of recording and transmission. The original images are often contaminated by a mixture of Gaussian and impulse noise. In this case, conventional image restoration methods using the Wiener filter or the Kalman filter [1] usually produce poor results. Several robust image restoration methods have been proposed. For example, Kashyap and Eom [2] developed a robust technique for estimating image model parameters, and Belaifa and Schwarty [3] proposed a robust method for estimating the original image intensity by using a reduced updated Kalman filter (RUKF). However, these methods perform well only where there is a very low probability of impulse noise.

Various forms of nonlinear techniques have also been introduced to solve the problem [4–11]. Among them, the median filter seems to offer better performances in terms of preserving edge information and removing impulse noise. However, one of the problems of the median filter is its fixed window size which limits its performance. A large window leads to good impulse noise suppression but tends to smooth the whole image, whereas a small window cannot adequately remove the noise. More seriously, the median filter also destroys fine details, and produces streaks and blotches in restored images. Its variants, the centre weighted median filter (CWMedian) [5] and the multistage median filter [4, 6], improve performance. However, they still fail to suppress impulse noise effectively while preserving details of the image, especially where there is a high probability of impulse noise.

In this paper we propose a technique called the recursive minimum–maximum method, for removing impulse noise in images. It involves two steps: impulse detection and nonlinear filtering. It optimally uses neighbourhood information and works effectively even when there are high probabilities of impulse noise. This proposed technique can act as a preprocessor and be combined with other restoration techniques for blurred images.

The minimum–maximum method is introduced below, followed by a discussion on its statistical properties. Results on applying the method to noisy images are also presented. Throughout this paper, comparative studies are carried out between the proposed technique and the median filters [4], the CWMedian filter [5] and the multistage median filter [4].

2 Minimum–maximum method

In the following discussion, the noise model is given by

$$\xi(i,j) = v(i,j) \quad \text{with probability } \beta \qquad (1)$$

where $v(i, j)$ is an outlier process and β is a constant. The outlier may take on one of three forms: positive impulse noise, negative impulse noise, and combined positive and negative impulse noise (also called salt–pepper noise). A pixel corrupted by outlier noise appears as either a white or a black spot in the image [4]. This means that the value of the outlier pixel is usually much larger or smaller than its neighbours. The motivations of the recursive minimum–maximum method are to reduce unnecessary blurring, and to optimally make use of neighbourhood information around the detected outlier to estimate the grey level of the pixel and replace it with the estimation. It consists of two steps:

(i) detection of outlier pixels

(ii) estimation of the original grey levels of the blurred pixels.

2.1 Outlier detector

The first step is the detection of outliers. A simple impulse detector determines whether any intensity value is at one of the two extremes of its local intensity

IEE Proceedings online no. 19981995

Paper first received 18th November 1996 and in revised form 24th March 1997

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distribution. However, this method does not work when an impulse appears in a high gradient image. In addition, it may not work when the impulse is more than one element wide. Nevertheless, we can still make use of the simple characteristics of the outlier, i.e. it appears as either a white or black spot in the image.

Gonzalez [9] introduced the point detector that computes the difference between the value of a pixel and the average value of its neighbours. That particular pixel is classified as an outlier if this difference is greater than a threshold value. A suitable choice of the threshold is critical. Generally speaking, the point detector may fail to recognise an impulse noise if the threshold is set too high. On the other hand, it may misclassify a legitimate pixel as an outlier if the threshold is set too low.

d_1	d_2	d_3
d_4	d_9	d_5
d_6	d_7	d_8

Fig. 1 *Window used to process and detect outliers*

In our method, we use the following algorithm for outlier detection.

1. Form a 3 × 3 window centred at the test pixel, as shown in Fig. 1.

2. Calculate D_i and S_{di}, where

$$D_i = d_i - d_9, \quad i = 1, \ldots, 8$$

$$S_{d_i} = sign(d_i - d_9), \quad i = 1, \ldots, 8, \text{ where}$$

$$sign(x) = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0 \end{cases}$$

3. Arrange the absolute values of D_i ($|D_i|$) in ascending order of magnitude to obtain $\{\bar{D}_{i:8}, i = 1, \cdots, 8\}$. Then calculate

$$\Delta D = \frac{\bar{D}_{4:8} + \bar{D}_{5:8}}{2}$$

4. A pixel value is classified an outlier if $(\Delta D > T) \wedge (\Sigma S_{di} = -8 \vee \Sigma S_{di} = 8)$. Otherwise, consider it as part of local image structure. Here T is a non-negative threshold.

In our outlier detector, we have introduced the necessary condition that an outlier must either be larger or smaller than the value of any of its neighbouring pixels. The value of the threshold establishes the relative difference in grey level that classifies the pixel under consideration as an outlier. (The effects of the choice of threshold on a restored image are discussed in Section 2.3.)

2.2 Minimum–maximum estimator

When an outlier is detected, the minimum–maximum estimator is then used to estimate the grey level of the corrupted pixel, based on neighbourhood information. The algorithm for this estimator is as follows.

1. Form the sets

$L = \{L_i, i = 1, \cdots, 4\}$ where $L_i = max(d_i, d_{9-i})$

$E = \{E_i, i = 1, \cdots, 4\}$ where $E_i = min(d_i, d_{9-i})$

2. Set

$P_{max} = max(E_1, \cdots, E_4)$

$P_{min} = min(L_1, \cdots, L_4)$

3. Replace the outlier pixel value by

$$y = \frac{P_{max} + P_{min}}{2} \tag{2}$$

Note that if there are three identical outliers along one direction within the window, then the output of the minimum–maximum estimator is largely influenced by the outlier noise. In this case, either P_{max} or P_{min} is equal to the level of outlier noise.

However, in the detection window shown in Fig. 1, $\{d_1, d_2, d_3, d_4\}$ are, in practice, the previous outputs of the filter, instead of the original degraded image data. Thus, the output of the recursive minimum–maximum estimator is derived from the last four outputs and the present five inputs in the window. Based on our observations, the recursive minimum–maximum method introduces a very small amount of blurring into the restored image but suppresses the impulse noise effectively.

The statistical properties of the minimum–maximum method are discussed in Section 3. The method also has the following properties which are trivial to prove.

(i) It is scaling and translation invariant; i.e. if $y(I, j)$ is the output produced by the method for input $x(I, j)$, then for input $k_1x(I, j) + k_2$ the output is $k_1y(I, j) + k_2$.

(ii) It preserves a constant signal.

(iii) It preserves step edges and ramps.

2.3 Choice of threshold values for the outlier detector

The threshold T in the outlier detector has a certain relationship with the attributes of the image and the outlier occurring probability. Two images are chosen as examples to investigate this phenomenon; 'baboon' and 'pepper', as shown in Fig. 2. The 'baboon' image has a lot of fine details, whereas the 'pepper' image has more smooth regions and step regions.

The recursive minimum–maximum method is used to process the images that are corrupted by outlier noise. The effect of the choice of threshold value on the mean square error (MSE) of the output of the recursive minimum–maximum method is examined. This effect is studied with different noise probabilites. The results are shown in Figs. 3 and 4. They show that a threshold value between 30 and 45 gives better performance in all situations. Moreover, the MSE values are still acceptable even with a threshold $T = 0$.

3 Statistical properties of minimum–maximum method

The robustness of an estimator subject to impulse noise is indicated by the breakdown probability. The breakdown probability is the probability of an impulse occurring at the output of an estimator. In this Section, the breakdown probability of the minimum–maximum method is derived. This is compared with the median filters and its variants; the CWMedian filter [5] and the multistage median filter (bidirectional) [6].

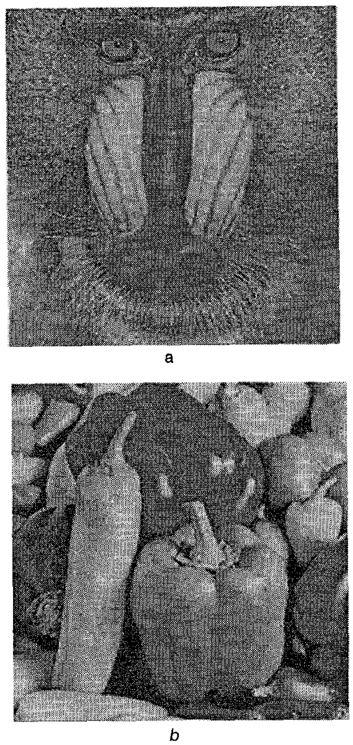

Fig. 2 *Images for investigating thresholds for the recursive minimum–maximum method*
a 'Baboon'
b 'Pepper'

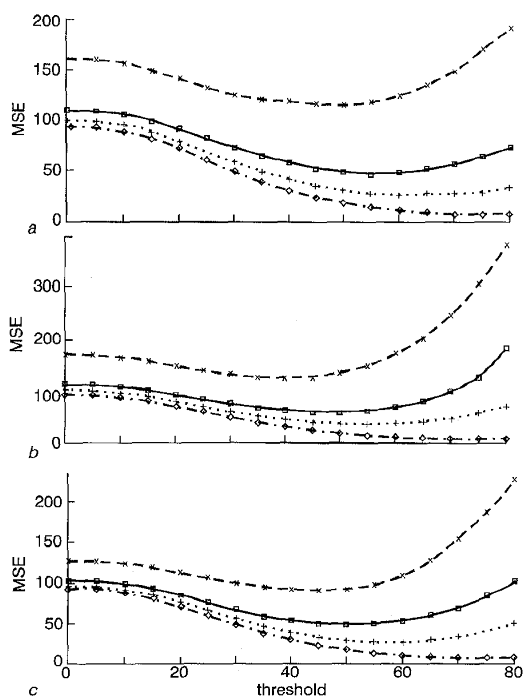

Fig. 3 *Variation of MSEs of recursive minimum–maximum method with respect to the threshold and probability of impulse noise for 'baboon' image*
—×— probability = 0.2
—□— probability = 0.1
···+··· probability = 0.05
–·◇–· probability = 0.01
a With positive impulse noise
b With negative impulse noise
c With salt–pepper noise

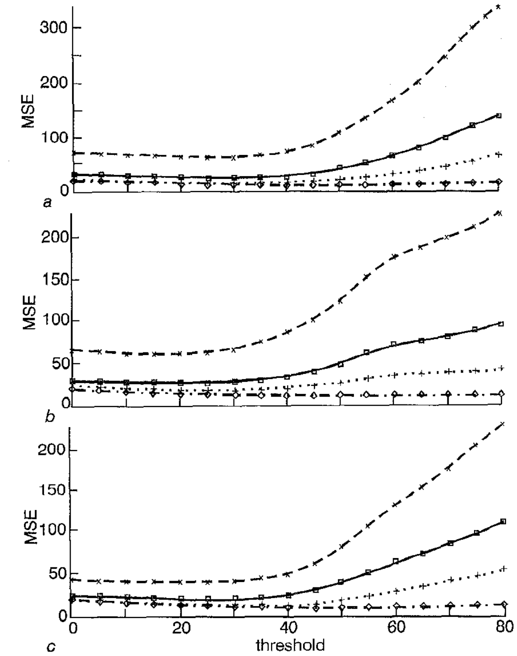

Fig. 4 *Variation of MSEs of recursive minumum–maximum method with respect to the threshold and probability of impulse noise for 'pepper' image*
—×— probability = 0.2
—□— probability = 0.1
···+··· probability = 0.05
–·◇–· probability = 0.01
a With positive impulse noise
b With negative impulse noise
c With salt–pepper noise

Let $\{x(\mathbf{n}): (\mathbf{n}: (i, j) \in (0, ..., N-1))\}$ be an independent identically distributed (i.i.d) discrete random sequence with probability measure function $F_x(j)$. $x(\mathbf{n})$ is quantised to one of the k values $(0, 1, ..., k-1)$, i.e k-level quantisation.

For the minimum–maximum method, the probability measure functions F_L and F_S of the sets L and S, respectively, are as follows:

$$\begin{aligned} F_L(j) &= P\{\max(d_i, d_{9-i}) < j\} \\ &= P\{d_i < j\}\,P\{d_{9-i} < j\} \\ &= P^2\{x(\mathbf{n}) < j\} = F_x^2(j) \end{aligned} \qquad (3)$$

$$\begin{aligned} F_S(j) &= P\{\min(d_i, d_{9-i}) < j\} \\ &= 1 - P\{\min(d_i, d_{9-i}) > j\} \\ &= 1 - P\{d_i > j\}\,P\{d_{9-i} > j\} \\ &= 1 - P^2\{x(\mathbf{n}) > j\} \\ &= 1 - \{1 - P\{x(\mathbf{n}) < j\}\}^2 \\ &= 1 - [1 - F_x(j)]^2 \end{aligned} \qquad (4)$$

The probability measure function $F_{max}(j)$ of P_{max} is given by

$$F_{max}(j) = P\{\max(S_1, \ldots, S_4) < j\} \qquad (5)$$

Since $\{x(\mathbf{n})\}$ is i.i.d, it is obvious that $\{S_i\}$ is also i.i.d. Therefore

$$\begin{aligned} &F_{max}(j) \\ &= P\{S_1 < j\}\,P\{S_2 < j\}\,P\{S_3 < j\}\,P\{S_4 < j\} \\ &= F_S^4(j) = \left\{1 - [1 - F_x(j)]^2\right\}^4 \end{aligned} \qquad (6)$$

Similarly, the probability measure function $F_{min}(j)$ of

P_{min} is

$$\begin{aligned} F_{min}(j) &= P\{\min(L_1, \ldots, L_4) < j\} \\ &= 1 - P\{\min(L_1, \ldots, L_4) < j\} \\ &= 1 - P(L_1 > j)P(L_2 > j)P(L_3 > j)P(L_4 > j) \\ &= 1 - [1 - F_L(j)]^4 = 1 - \left[1 - F_x^2(j)\right]^4 \end{aligned} \quad (7)$$

Now if p is the probability of an impulse occurring at the input, then the breakdown probability of the maximum part P_{rmax} and the breakdown probability of the minimum part P_{rmin} are

$$P_{max} = \left\{1 - [1-p]^2\right\}^4 = [p(2-p)]^4 \quad (8)$$

$$P_{min} = 1 - \left[1 - p^2\right]^4 \quad (9)$$

According to eqn. 2, the breakdown probability P_r for the minimum–maximum method is

$$\begin{aligned} P_r &= P_{rmax} \cdot P_{rmin} \\ &= p^4(2-p)^4\left[1 - (1-p^2)^4\right] \end{aligned} \quad (10)$$

The breakdown probabilities of the median filter and its variants (CWMedian filter and multistage median filter) with the probability of impulse occurrence p are given by [5, 6, 11]

median filter:

$$P_r = \sum_{q=L+1}^{2L+1} \binom{2L+1}{q} p^q (1-p)^{2L+1-q} \quad (11)$$

where $2L + 1 = (2N + 1)(2N + 1)$ and $2N + 1$ is the window size.

CWMedian filter:

$$\begin{aligned} P_r = &\sum_{q=k_1-1}^{2L} \binom{2L}{q} p^{q+1}(1-p)^{2L-q} \\ &+ \sum_{q=k_2}^{2L} \binom{2L}{q} p^q (1-p)^{2L+1-q} \end{aligned} \quad (12)$$

where $k_1 = L + 1 - k$, $k_2 = L + 1 + k$ and k is the number of centre weight.

Multistage median filter:

$$\begin{aligned} P_r = p&\left\{1 - \left[\sum_{q=2N+1}^{4N} \binom{4N}{q} (1-p)^q p^{4N-q}\right]^2\right\} \\ &+ (1-p)\left[\sum_{q=2N+1}^{4N} \binom{4N}{q} (1-p)^{4N-q} p^q\right]^2 \end{aligned} \quad (13)$$

Table 1 compares the numerical values of the breakdown probabilities of the above methods for different values of p and different window sizes. It shows that with a larger window (5 × 5 or larger) the median filter, the CWMedian filter and the multistage median filter all give a better performance in suppressing impulse noise. This is not suprising since the larger the window the more data are involved. However, also note that with a larger window these filters destroy the fine details in the image and the computational requirements are higher. When the input probability p is higher, the minimum–maximum method performs better than the other filters.

Note that these breakdown probabilities are for the non-recursive version of the minimum–maximum method. Better results should be expected when using the recursive method. This point is illustrated by the empirical results discussed below. The breakdown probability expression for the recursive minimum–maximum method can be obtained by using a similar method as that described above. However, in this case, the probability measure function of $\{d_i, i = 1, \cdots, 4\}$ is no longer $F_x(j)$ but the last recursive output probability measure function.

4 Simulation results

Two simulation experiments were undertaken to demonstrate the performance of the proposed recursive minimum–maximum method. Comparisons are made with the median filters, the CWMedian filter and the multistage median filter (bidirectional). To provide consistent comparison, only the recursive versions of these filters are used. The objective quantitative measure used for comparison is the MSE between the original and restored images, defined by

$$MSE = \frac{1}{N^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} [f(\mathbf{n}) - y(\mathbf{n})]^2 \quad (14)$$

where $\{f(\cdot)\}$ and $\{y(\cdot)\}$ are the original and restored images, respectively.

The first experiment is designed to study how well the various filters preserve structural information in the image while removing impulse noise. It is assumed that the impulse noise consists of impulses at a fixed (known) grey level. The filters only process those pixels that are detected as outliers. The 'baboon' and 'pepper' images are used as test images.

Table 2 shows the statistical MSEs of the restored images, which are the MSEs averaged over a number independent trials. These results show that the median filter with a smaller window is better at preserving structural information, but fails to suppress the impulse noise when the probability of impulse occurrence is high. The median filter with a larger window has opposite characteristics, which agrees with previous theoreti-

Table 1: Breakdown probabilities of median filters and their variants for different values of p

Probability	Median (3 × 3)	Median (5 × 5)	CWMedian (5 × 5, 2k + 1 = 5)	CWMedian (5 × 5, 2k + 1 = 7)	Multistage median (5 × 5)	Multistage median (7 × 7)	Minimum–maximum
0.0625	9.7×10^{-5}	5.64×10^{-10}	4.93×10^{-8}	5.0×10^{-7}	1.09×10^{-4}	4.94×10^{-6}	3.34×10^{-6}
0.1	8.90×10^{-4}	1.60×10^{-7}	5.21×10^{-6}	3.21×10^{-5}	1.00×10^{-3}	1.08×10^{-4}	5.14×10^{-5}
0.2	1.95×10^{-2}	3.69×10^{-4}	2.52×10^{-3}	7.23×10^{-3}	2.20×10^{-2}	7.70×10^{-3}	2.53×10^{-3}
0.3	9.88×10^{-2}	1.75×10^{-2}	4.65×10^{-2}	8.26×10^{-2}	1.07×10^{-1}	6.70×10^{-2}	2.13×10^{-2}
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.216

Table 2: MSEs of recursive median filters and their variants for 'baboon' and 'pepper' images with the outliers considered as impulses at some fixed and known locations

Probability	Noise style	Median (3×3)		Median (5×5)		CWMedian (5×5)		Multistage median (5×5)		Minimum–maximum	
		'baboon'	'pepper'	'baboon'	'pepper'	'baboon'	'pepper'	'baboon'	'pepper'	'baboon'	'pepper'
0.1	salt–pepper	2.19	0.88	3.10	2.94	5.09	4.53	4.81	3.91	2.02	0.97
	positive impulse	2.00	0.69	3.70	2.75	4.94	3.83	4.83	3.15	2.12	0.94
0.0625	salt–pepper	15.23	8.15	21.40	9.66	31.14	21.86	29.28	19.49	13.76	5.13
	positive impulse	19.07	16.88	21.56	12.27	33.09	30.24	32.33	27.82	13.36	5.87
0.1	salt–pepper	27.31	16.25	23.86	13.51	52.33	39.09	49.88	34.59	22.70	8.92
	positive impulse	52.92	32.89	35.56	18.58	56.22	43.13	54.99	36.98	23.49	10.88
0.2	salt–pepper	98.41	58.99	71.94	36.43	99.44	77.31	100.3	72.96	53.10	25.77
	positive impulse	371.60	334.15	80.20	60.72	148.4	146.45	169.5	148.83	62.07	35.70
0.5	salt–pepper	1451	1791	271	204	378	381	456	475	245	188.1
	positive impulse	7390	8543	719	1039	8373	9473	4404	5004	1040	1338

Table 3: MSEs of recursive median filters and their variants for 'baboon' and 'pepper' images compared with the recursive min–max method

Probability	Noise style	Median (3×3)		Median (5×5)		CWMedian (5×5)		Multistage median (5×5)		Minimum–maximum	
		'baboon'	'pepper'	'baboon'	'pepper'	'baboon'	'pepper'	'baboon'	'pepper'	'baboon'	'pepper'
0.1	salt–pepper	154.2	32.1	285.3	91.8	190.1	57.7	184.3	62.6	30.78	8.40
	positive impulse	155.8	33.1	285.1	90.9	188.1	56.2	182.0	61.0	30.55	8.07
0.0625	salt–pepper	165.4	47.4	288.1	104.5	208.85	75.9	206.44	95.0	43.35	21.10
	positive impulse	175.9	55.1	294.7	116.3	207.0	81.5	208.8	97.4	44.19	14.30
0.1	salt–pepper	184.1	60.2	300.7	112.7	224.7	93.4	228.1	112.1	54.78	23.48
	positive impulse	220.8	88.9	313.5	146.4	226.8	102.6	236.2	127.1	58.51	24.23
0.2	salt–pepper	255.6	130.0	232.4	140.9	263.1	133.8	276.5	173.0	94.59	40.31
	positive impulse	726.8	611.0	434.4	300.9	347.5	217.0	377.8	297.0	120.14	70.96
0.5	salt–pepper	2260	2296	573	469.9	482	452.0	583	597.3	368.68	313.4
	positive impulse	11591	11517	11903	12970	12783	13680	6843	7638	5493	5536

Table 4: MSEs of non-recursive median filters and their variants for 'baboon' and 'pepper' images compared with the recursive min–max method

Probability	Noise style	Median (3×3)		Median (5×5)		CWMedian (5×5)		Multistage median (5×5)		Minimum–maximum	
		'baboon'	'pepper'	'baboon'	'pepper'	'baboon'	'pepper'	'baboon'	'pepper'	'baboon'	'pepper'
0.1	salt–pepper	154.4	33.00	284.3	92.00	135.4	37.86	133.0	48.16	30.78	8.40
	positive impulse	154.7	32.90	284.6	91.73	134.7	37.63	133.2	46.30	30.55	8.07
0.0625	salt–pepper	167.4	48.06	289.7	105.0	152.8	53.96	157.8	79.86	43.35	21.10
	positive impulse	176.2	56.50	294.1	114.8	154.4	62.33	165.2	89.66	44.19	14.30
0.1	salt–pepper	179.2	59.56	295.3	111.8	164.1	68.33	178.3	99.46	54.78	23.48
	positive impulse	219.8	100.0	313.5	146.4	180.0	91.43	212.2	141.2	58.51	24.23
0.2	salt–pepper	257.8	139.5	318.9	144.0	208.7	116.3	263.4	186.3	94.59	40.31
	positive impulse	728.6	598.3	430.5	300.9	468.9	381.4	624.4	589.3	120.14	70.96
0.5	salt–pepper	2167	2253	570.5	461.0	1560	1605	1572	1870	368.68	313.4
	positive impulse	11666	12618	12035	12943	11717	12773	9623	10645	5493	5536

cal analysis. The CWMedian filter and the multistage median filter have the same problem. However, the recursive minimum–maximum method is able not only to preserve the structural information well, but also suppress the impulse noise effectively even when the probability of impulse occurrence is high. Table 4: MSEs of non-recursive median filters and their variants for 'baboon' and 'pepper' images compared with the recursive min–max method.

In the second experiment, the same test images are used. However, the outlier consists of impulses at unknown grey levels. This is the situation often encountered in practice. Here, two groups of simulations are performed. The first group is with the recursive median filters and its variants, and the other group is with the non-recursive version of these filters. It is obvious that the recursive median filters and its variants give a better performance in suppressing impulse noise, especially in the case of the high probability of outliers. They also tend to blur the fine details. On the other hand, the non-recusive median filters and its variants give a better performance in terms of preserving

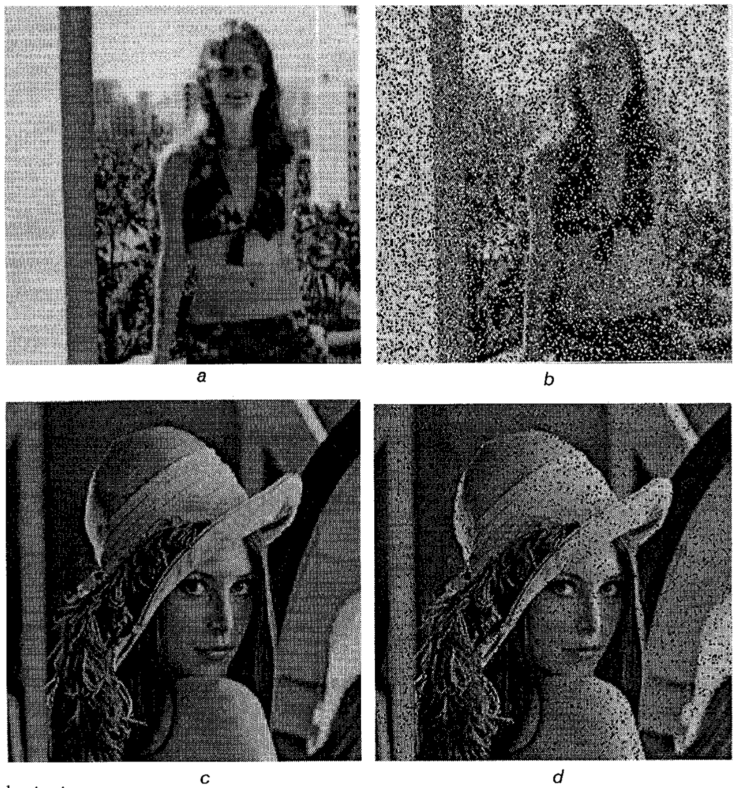

Fig. 5 *The original image and noisy images*
a Original 'Diane' image; *b* 'Diane' image blurred by salt–pepper noise (p = 0.3)
c Original 'Lena' image; *d* 'Lena' image blurred by negative impulse noise (p = 0.0625)

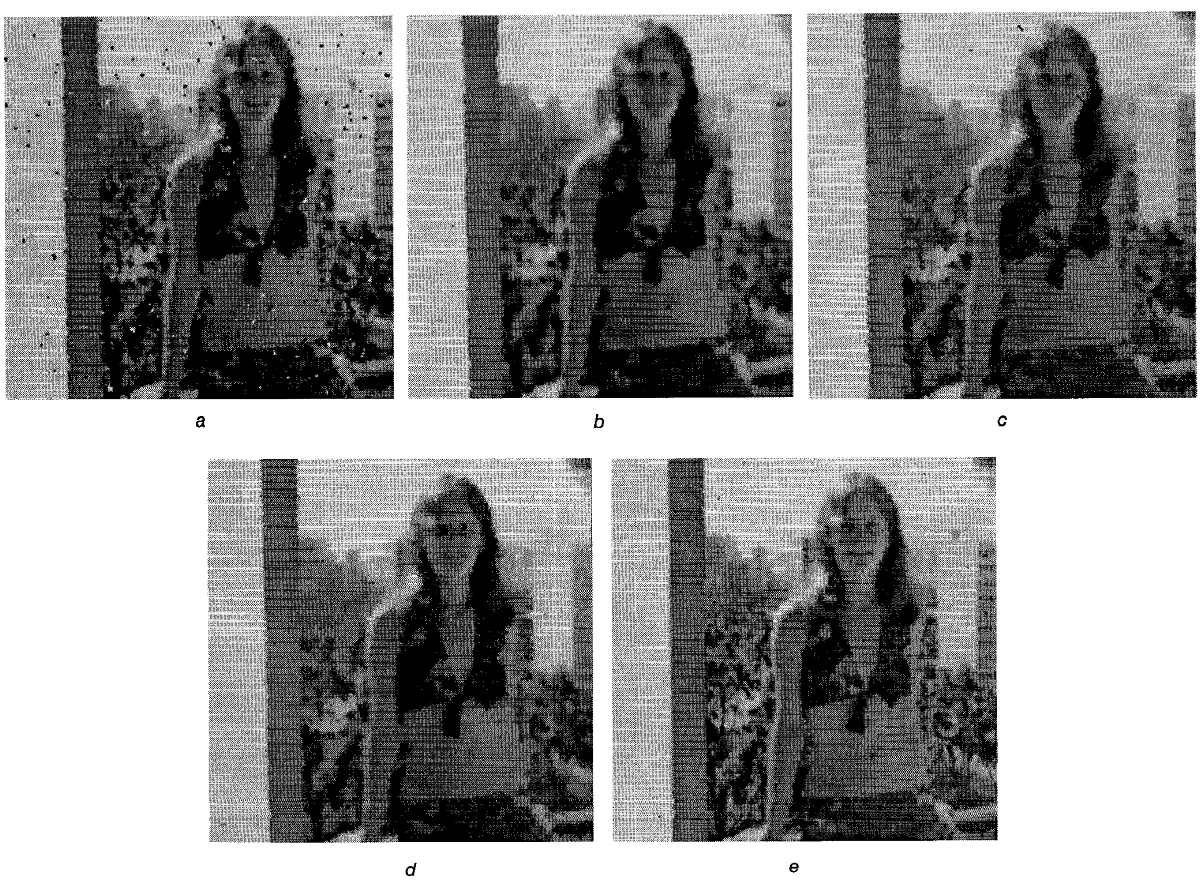

Fig. 6 *Restored 'Diane' images*
a Recursive median filter (3 × 3); *b* Recursive median filter (5 × 5); *c* Recursive multistage filter (5 × 5)
d Recursive CWMedian filter (5 × 5); *e* Recursive minimum–maximum method

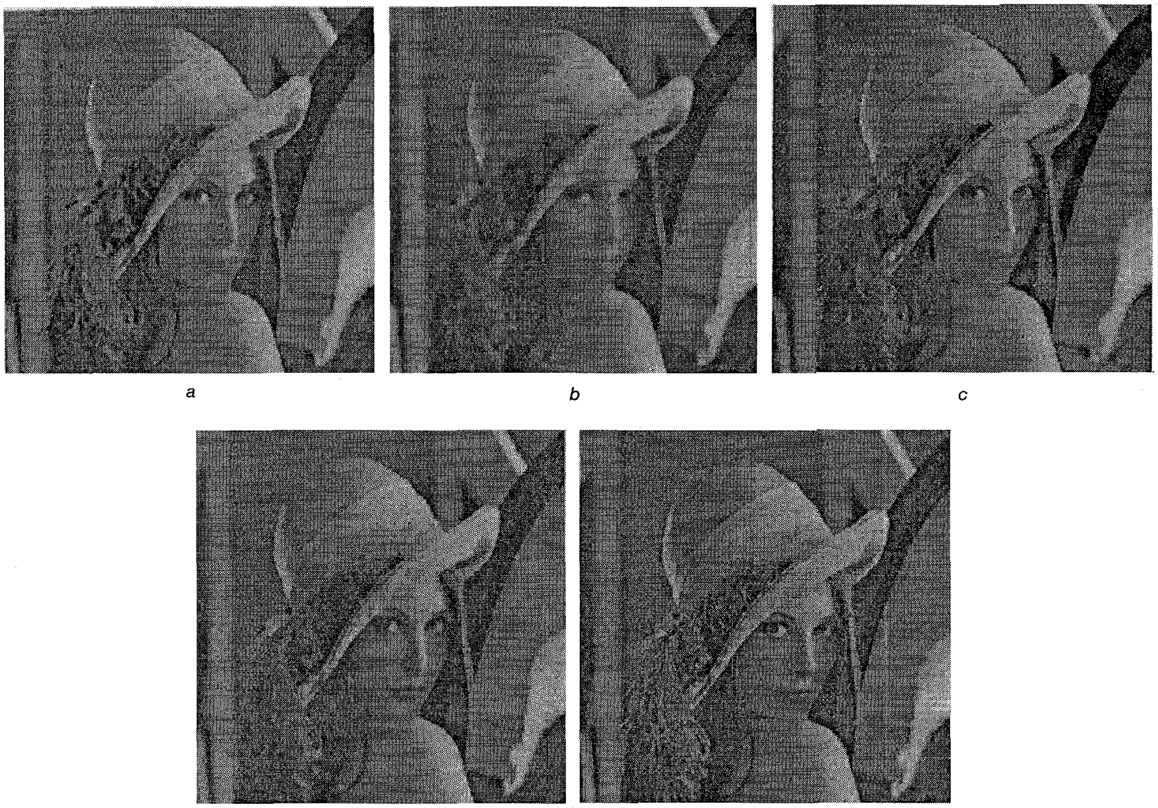

Fig. 7 *Restored 'Lena' images*
a Nonrecursive median filter (3 × 3); *b* Nonrecursive median filter (5 × 5); *c* Nonrecursive multistage filter (5 × 5)
d Nonrecursive CWMedian filter (5 × 5); *e* Recursive minimum–maximum method

edge and fine details in the restored image, but cannot suppress the impulse noise effectively when the probability of impulse occurrence is high. The statistical experimental results are listed in Tables 3 and 4 for the first and second group of filters, respectively. The recursive minimum–maximum method results in the smallest MSE and is robust, in that it maintains its performance across different images and different types of outlier.

The effectiveness of the recursive minimum–maximum method is further illustrated by restoring two other images with high and low probability of impulse occurrence. Figs. 5*a* and *c* are the original 'Diane' and 'Lena' images. Figs. 5*b* and *d* are their corresponding degraded images corrupted by salt–pepper noise with $p = 0.3$, and by negative impulse noise with $p = 0.0625$, respectively. Since the recursive median filters and its variants give a better performance in terms of MSE in the case of the high probability of outliers, they are used to restore the image in Fig. 5*b*. The non-recursive version of median filters and its variants are used to restore the image in Fig. 5*d*. The restored images are shown in Figs. 6 and 7. These results show again that our proposed method works effectively under both high and low probability of impulse occurrence.

5 Conclusions

In this paper, we have introduced the recursive minimum–maximum method for removing outlier noise from images. This method can remove the outliers and preserve fine details effectively under both high and low probabilities of impulse occurrence. As shown by illustrative examples, the performance of the proposed method is better than that of median filters and its variants, both recursive and non-recursive. The proposed method can be used as a pre-processor, which can be combined with the other image restoration techniques to enhance the robustness to impulse noise.

6 References

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